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THE THEORY OF EVOLUTION AND DYNAMICAL SYSTEMS

Mathematical Aspects of Selection

(London Mathematical Society Student Texts 7)

By J. HOFBAUER and K. SIGMUND: pp. 341. £35.00/£11.95. LMS members' price £26.25/£8.96. (Cambridge University Press, 1988)

Not many years ago, a text describing the non-statistical applications of mathematics in biology would have been a slim volume: predator-prey equations and a smattering of population genetics. But now the authors of the original German book, published in 1984, present an updated English account of the mathematical treatment of deterministic models inspired by biology, that amply fills 320 pages at the level of a mathematics graduate student. Much of the material, especially in the second half of the book, was developed by one or other of the authors and their collaborators.

There are brief accounts of the real biology behind the models, and references to more extensive sources. Particularly striking is the wide context within which the so-called replicator equation

$$\dot{x}_i = x_i(f_i(\mathbf{x}) - \bar{f}(\mathbf{x})), \quad i = 1, 2, \ldots, n,$$

where $\mathbf{x} \in S_n = \{\mathbf{x} = (x_1, \ldots, x_n), x_i \geqslant 0, \Sigma x_i = 1\}$ and $\bar{f} = \Sigma x_i f_i$, arises as a plausible model. It is found in the Lotka–Volterra competition models in ecology, the hypercycles of Eigen and Schuster in prebiotic evolution, the game dynamics of Taylor and Jonker, and the mutation-selection processes of population genetics. In the first half of the book, $f_i(\mathbf{x}) = \Sigma a_{ij} x_j$, and the main mathematical ideas used are Lyapunov functions, stability of solutions of ODEs, and permanence. (A dynamical system on S_n is permanent if there is some $\delta > 0$ so that $\liminf_{t\to\infty} x_i(t) > \delta$ for all i whenever $x_i(0) > 0$ for all i.)

The second half, although most rewarding, is not an easy read. Much of the work has arisen only in the past ten years, and the notation is too often unhelpful or unmemorable. For example, the symbol for mutation changes from the usual μ in the first part to ε in the second, and we meet B-matrices, M-matrices, P-matrices, c-partnerships, and D-stability. The separate chapters seem less coherent than earlier and, as all labelled results are honoured as 'Theorems', picking out those of greatest importance requires some guesswork.

The Exercises are an important feature. Some are there to give practice in applying the results to numerical examples, and others indicate alternative proofs of Theorems. But many of them tidy up loose ends, generalize, or point to related work in the published literature. The Preface warns that some of them are quite hard, and conscientious readers will be kept on their toes. If anything, the Exercises play a larger role later in the book than earlier.

In summary, this is a timely and authoritative account of the use of mathematical models to explore aspects of biology. My reservations are minor: the notation would be easier to live with if a glossary were supplied, and I would welcome both a fuller index and a set of Answers to (at least) the routine Exercises. But we have here a lively and wide-ranging book of applicable mathematics, seeking answers to the fundamental questions of how did the living world evolve, and how does it survive.

JOHN HAIGH