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CHAIN RECURRENCE AND DISCRETISATION

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Upper and lower semicontinuity results for the chain recurrent set are shown to remain valid in numerical dynamics with constant stepsizes. It is also pointed out that the chain recurrent set contains numerical ω–limit sets for discretisations with a variable stepsize sequence approaching zero.

1. INTRODUCTION

We consider an autonomous differential equation

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n \tag{1}$$

where f is of class C^2, f' bounded: $|f'| \leqslant \gamma_0$.

Consider a C^2 discretisation method φ of order $p \geqslant 1$, which means there exists a constant K (depending only on f) such that

$$|\varphi(h,x) - \Phi(h,x)| \leqslant Kh^{p+1} \qquad \text{for all} \quad h \in [0,h_0] \quad \text{and} \quad x \in \mathbb{R}^n \tag{2}$$

where $\Phi(t,x)$ denotes the flow defined by (1).

The differentiability assumption on φ implies that $\varphi(h,\cdot)$ is a C^2 diffeomorphism of $\mathbb{R}^n$ onto itself (it is enough to assume that the mixed partial derivative φ_{hx} is bounded on $[0,h_0] \times \mathbb{R}^n$; see Garay [**8**, Remark 2.4]). Hence the map $\varphi(h,\cdot)$ defines a discrete time dynamical system on $\mathbb{R}^n$.

A simple application of Gronwall's lemma to the variational equation yields the estimate

$$|\Phi_x(h,x)| \leqslant 1 + \gamma h \qquad \text{for all} \quad h \in [0,h_0] \quad \text{and} \quad x \in \mathbb{R}^n \tag{3}$$

where the constant γ can be chosen arbitrarily close to γ_0 as $h_0 \to 0$.

Assume that $\Omega = \{\infty\}$ is a repeller for the flow Φ, that is, the system is 'dissipative', and there is a dual attractor $\mathcal{A}$. We assume for simplicity, that Ω is a repellor also for the diffeomorphism $\varphi(h,\cdot)$ for small h. (This assumption could be relaxed by working on the one point compactification sphere. Everything, we discuss in this paper, can be done on compact manifolds. The necessary technique needed to lift the results from $\mathbb{R}^n$ to the one point compactification sphere or to general compact manifolds is described in Garay [**9**].) For basic concepts of dynamics we use in this paper, see for example, Irwin [**10**].

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2. Upper semicontinuity of the chain recurrent set

It is well-known, see Stuart [**13**, section 6] or originally Kloeden and Lorenz [**11**], that the iteration of $\varphi(h,\cdot)$ has an attractor $\mathcal{A}_h$ which, as $h \to 0$, goes to $\mathcal{A}$ in an upper semicontinuous way, that is, $\limsup_{h\to 0} \mathcal{A}_h = \bigcap_{h>0} \overline{\bigcup_{\tau<h} \mathcal{A}_\tau} \subseteq \mathcal{A}$, or equivalently, for all $\varepsilon > 0$, $\mathcal{A}_h \subseteq \mathcal{N}_\varepsilon(\mathcal{A})$, as $h \to 0$, where $\mathcal{N}_\varepsilon(\mathcal{A})$ denotes the ε-neighbourhood of $\mathcal{A}$ in $\mathbb{R}^n$. In this section we prove that the same holds for the chain recurrent set.

We use the following theorem, which combines a result of Conley with the smoothing method of Wilson, see for example, Akin [**1**, Theorem 6.12].

Theorem 1. *Let (A,R) be an attractor-repeller pair for the flow. Then there exists a Liapunov function $V : \mathbb{R}^n \to [0,1]$ of class C^∞ such that $V^{-1}(0) = A$, $V^{-1}(1) = R$, and*

$$\dot V(x) = \langle \operatorname{grad} V(x), f(x)\rangle < 0 \qquad \text{for all} \quad x \in U = \mathbb{R}^n \setminus (A \cup R). \tag{4}$$

There is no loss of generality in assuming that V is globally Lipschitz with constant L.

Lemma 1. *For $c \in (0,1)$ there exists an $h^* = h^*(c)$ such that*

$$V(x) \leqslant c \Rightarrow V(\varphi(h,x)) < c \quad \text{for} \quad 0 < h < h^*$$

Proof: We distinguish two cases according as $V(x) \leqslant c/2$ or not. If $V(x) \leqslant c/2$ then we show $V(\varphi(h,x)) < c$. This follows for small h from the estimate

$$\begin{aligned} V(\varphi(h,x)) &\leqslant V(x) + |V(\varphi(h,x)) - V(\Phi(h,x))| + V(\Phi(h,x)) - V(x) \\ &\leqslant \frac{c}{2} + LKh^{p+1} + 0. \end{aligned} \tag{5}$$

If $c/2 \leqslant V(x) \leqslant c$ then we show $V(\varphi(h,x)) < V(x)$ for sufficiently small h. Note first that the set $\{\Phi(\tau,x) : \tau \in [0,h_0] \text{ and } c/2 \leqslant V(x) \leqslant c\}$ is compact. Then apply (5) again, by observing that now the last term is equal to

$$\int_0^h \dot V(\Phi(\tau,x))d\tau$$

where the integrand is negative by (4), and hence smaller than some constant $-\alpha = -\alpha(c) < 0$. So (5) is

$$\leqslant V(x) + LKh^{p+1} - \alpha h < V(x)$$

for h sufficiently small. ∎

Lemma 1 implies the existence (and its upper semicontinuity) of the numerical attractor A_h, by

$$A_h = \bigcap_{N=0}^{\infty} \overline{\bigcup_{M=N}^{\infty} \{\varphi^M(h,x) : V(x) \leqslant c\}},$$

see for example, Stuart [**13**, Theorem 6.2]. (Observe that $A_h = \mathcal{A}_h$ if $R = \Omega = \{\infty\}$.) Since $\varphi(h,\cdot)$ is a diffeomorphism, we can reverse time, and conclude the existence of the dual repellor R_h. For the upper semicontinuity of R_h we have to show $R_h \subseteq \{x : V(x) > 1-c\}$ for small h, which follows from the next Lemma.

Let $U_h = \mathbb{R}^n \setminus (A_h \cup R_h)$ denote the set of connecting (or transient) orbits.

Lemma 2. *For $c \in (0,1/2)$ there exists an $h^{**} = h^{**}(c)$ such that*

$$U_h \supset V^{-1}[c, 1-c] \quad \textit{for} \quad 0 < h < h^*.$$

Proof: By compactness, there is a uniform time $T = T(c)$, such that

$$\Phi(t,x) \in V^{-1}[0, \tfrac{c}{2}] \quad \text{for all } t \geqslant T \quad \text{and } x \in V^{-1}[0, 1-c].$$

Applying the standard error estimate

$$\text{(6)} \qquad \left|\Phi(Nh,x) - \varphi^N(h,x)\right| \leqslant \kappa(T)h^p \qquad \text{for } h \leqslant h_0,\ x \in \mathbb{R}^n,\ 0 \leqslant Nh \leqslant T$$

with $\kappa(T) = K\left(e^{\gamma T} - 1\right)/\gamma$, we obtain for $N = [(T + h_0)/h]$

$$V\left(\varphi^N(h,x)\right) \leqslant V(\Phi(Nh,x)) + \left|V\left(\varphi^N(h,x)\right) - V(\Phi(Nh,x))\right| \leqslant \frac{c}{2} + L\kappa(T+h_0)h^p < c$$

for sufficiently small h and $x \in V^{-1}[0, 1-c]$. Hence the numerical approximation of orbits through those x enters the region $V^{-1}[0,c]$ and remains there by Lemma 1. ∎

The *chain recurrent set* is the intersection of all attractor—repeller pairs

$$C = \bigcap \{A^i \cup R^i : i \in I\}$$

for the ODE, and $C_h = \bigcap\{A_h^i \cup R_h^i : i \in I_h\}$ for the discretised dynamics. (The indexing sets I and I_h are at most countable.) These are equivalent to the standard definitions based on (ε, T)–chains. The concept of chain recurrence was introduced by Conley [**5**]; for a recent, comprehensive treatment we recommend Akin [**1**].

Recall and extend the notation $U^i = \mathbb{R}^n \setminus \left(A^i \cup R^i\right)$, $U_h^i = \mathbb{R}^n \setminus \left(A_h^i \cup R_h^i\right)$, and V_i for the Liapunov function from Theorem 1 for the attractor—repeller pair $\left(A^i, R^i\right)$.

The geometric meaning of the next result is that discretisation cannot make the chain recurrent set explode.

THEOREM 2. *One–step discretisation perturbs the chain recurrent set in an upper semicontinuous way, that is,*

$$\limsup_{h\to 0}\ \mathcal{C}_h \subseteq \mathcal{C}.$$

PROOF: It is enough to prove that any compact subset Q of $\mathbb{R}^n\backslash\mathcal{C}$ is also contained in $\mathbb{R}^n\setminus\mathcal{C}_h$.

The compactness of Q implies $Q \subseteq \bigcup\{U^{i_k} : k = 1,\cdots,N\}$ for suitable indices $i_k \in I$. Applying compactness again, there is a $c > 0$ such that

$$Q \subseteq \bigcup_{k=1}^{N} V_{i_k}^{-1}(c, 1-c) \subseteq \bigcup_{k=1}^{N} U_h^{i_k} = \mathbb{R}^n\backslash \bigcap_{k=1}^{N}\left(A_h^{i_k}\cup R_h^{i_k}\right) \subseteq \mathbb{R}^n\backslash\mathcal{C}_h,$$

where we used Lemma 2. ∎

Elementary examples show that Theorem 2 does not hold for other recurrence concepts, like the set of Auslander recurrent points, the nonwandering set, the Birkhoff centre, or the minimal centre of attraction. All these sets can explode by discretisation.

3. Lower Semicontinuity of the Chain Recurrent Set

The survey paper of Stuart [**13**] contains a structural assumption on (1) under which $\mathcal{A}_h$ goes to $\mathcal{A}$ in a lower semicontinuous way, as $h \to 0$, that is,

$$\liminf_{h\to 0}\ \mathcal{A}_h = \bigcap_{\varepsilon>0}\bigcup_{h>0}\bigcap_{0<\tau<h} \mathcal{N}_\varepsilon(\mathcal{A}_\tau) \supseteq \mathcal{A},$$

or equivalently, for each $\varepsilon > 0$, $\mathcal{N}_\varepsilon(\mathcal{A}_h) \supseteq \mathcal{A}$, as $h \to 0$. In this section we establish a lower semicontinuity discretisation result for the chain recurrent set. Note that our structural assumptions (7) and (8) are independent of that in Stuart [**13**]: No 'natural' modification of [**13,** Assumption 6.29] works for our purpose.

PROBLEM. Is lower semicontinuity of the chain recurrent set under discretisations a generic property of (1)? (The same question is open for attractors, too.) The corresponding perturbation results in Akin [1, Section 7] for diffeomorphisms and vector fields suggest that the answer is affirmative. The problem is that the discretised system is not a perturbation in the traditional sense. Since (7) below is satisfied if Φ is Morse–Smale, see for example, Irwin [**10**], the answer is affirmative in two dimensions. Note also that assumption (8) below may be close to genericity: recall the general density theorem of Pugh [**12**].

THEOREM 3. *Assume that the chain recurrent set of (1), where f is of class C^3, consists of finitely many hyperbolic equilibria P_q and finitely many hyperbolic periodic orbits Γ_p,*

$$\mathcal{C} = \bigcup\{\Gamma_p : p = 1,\dots r\} \cup \{P_q : q = 1,\dots s\}. \tag{7}$$

Then C is lower semicontinuous with respect to one–step discretisations, that is,

$$\liminf_{h\to 0} C_h \supseteq C.$$

PROOF: Consider first a hyperbolic equilibrium P from C. The numerical Hartman–Grobman lemma of Garay [**9**] implies the existence of positive constants Q and h_0 and of a closed neighbourhood $\mathcal{N}$ of P in $\mathbb{R}^n$, such that for all $h \in (0, h_0]$, there exists a hyperbolic fixed point $P_h \in \mathcal{N}$ of $\varphi(h,\cdot)$ with $d(P, P_h) \leqslant Qh^p$ (which is the maximal compact $\varphi(h,\cdot)$–invariant set in $\mathcal{N}$). Clearly $P_h \in C_h$.

Now consider the more difficult case of a hyperbolic periodic orbit Γ in C. By a result of Beyn [**3**, Theorem 2.1] (which requires the additional smoothness assumption), there exist positive constants Q and h_0 and a closed neighbourhood $\mathcal{N}$ of Γ in $\mathbb{R}^n$, such that for all $h \in (0, h_0]$, there exists a $\varphi(h,\cdot)$–invariant simple closed curve Γ^h which is the maximal compact $\varphi(h,\cdot)$–invariant set in $\mathcal{N}$ and has Hausdorff distance $d_H(\Gamma, \Gamma^h) \leqslant Qh^p$. As a special case of Garay [**7**, Theorem 4], Γ^h is of class C^2 (this is important for the Denjoy property), for $h \in (0, h_0]$.

Now we use some basic properties of circle diffeomorphisms, see for example, Irwin [**10**]. If the rotation number of $\varphi(h,\cdot)|_{\Gamma^h}$ is irrational, then the Denjoy property implies that $\Gamma^h \subset C_h$ since $\varphi(h,\cdot)|_{\Gamma^h}$ is conjugate to a rotation with dense orbits. If the rotation number is rational, then $\Gamma^h \subset C_h$ does not hold in general. But in this case Γ^h contains a periodic point π_h, whose (finite) orbit $\{\pi_h^k : k \in \mathbb{Z}\}$ is contained in C_h. From the estimate

$$\begin{aligned}\left|\pi_h^{k+1} - \pi_h^k\right| = \left|\varphi(h, \pi_h^k) - \pi_h^k\right| &\leqslant \left|\varphi(h, \pi_h^k) - \Phi(h, \pi_h^k)\right| + \left|\Phi(h, \pi_h^k) - \pi_h^k\right| \\ &\leqslant Kh^{p+1} + \text{const}\cdot h\end{aligned}$$

for $k \in \mathbb{Z}$, it follows easily that $\sup\{d(x, C_h) : x \in \Gamma\} \to 0$, as $h \to 0$. ∎

REMARKS.

An application of Theorems 2 and 3 to a problem from population genetics is given in [**4**].

COROLLARY. *In Theorem 3, assumption (7) can be replaced by:*

(8) *The set of hyperbolic equilibria and hyperbolic periodic orbits is dense in C.*

PROOF: There can be at most countably many hyperbolic equilibria and hyperbolic periodic orbits. Number them. Let H_n be the union of the first n of these hyperbolic objects. Since by assumption $\overline{\bigcup_{n=1}^{\infty} H_n} = C$, we have $d_H(H_n, C) \to 0$. Hence for each $\varepsilon > 0$ there is an n, such that $d_H(H_n, C) < \varepsilon/2$. To the finite union of hyperbolic objects H_n we can apply the arguments of the proof of Theorem 3, and get an h_0, such that for each $h \in (0, h_0]$ there is a corresponding $\varphi(h,\cdot)$–invariant set $H_n^h \subseteq C_h$ with $d_H(H_n, H_n^h) < \varepsilon/2$. Hence $d_H(H_n^h, C) < \varepsilon$ which implies the desired lower semicontinuity. □

The above condition includes the class of systems for which C has a hyperbolic structure, or equivalently Axiom A and the 'no cycles' condition holds, see [**6**]. These systems satisfy the 'Ω–stability theorem', or rather 'C–stability theorem', that is, the flow restricted to C is stable against small C^1–perturbations of f. It would be interesting to extend this result (in a somewhat weaker form, see the periodic orbit case above) to discretisations.

Some structural stability results of (1) with respect to discretisations (2) have been obtained by Garay [**9**].

4. Variable Stepsize

The discretisation method in (2) gives rise to one with variable stepsize sequence $(h_1, h_2, \cdots)$ by defining inductively $\varphi(h_m, \cdots, h_1; x) = \varphi(h_m, \varphi(h_{m-1}, \cdots, h_1; x))$, $\varphi(0; x) = x$, $0 < h_m \leqslant h_0$, $m = 1, 2, \cdots$.

We assume that $\lim\limits_{n\to\infty} h_n = 0$ and $\sum\limits_{n=1}^{\infty} h_n = \infty$. We can extend the sequence $\{\varphi(h_m, \dots, h_1; x)\}_{m=1}^{\infty}$ to a broken line Γ, that is, to a piecewise linear function $\Gamma : \mathbb{R}_+ \to \mathbb{R}^n$ with kinks at the points $\sum h_i$, by setting $\Gamma(h_m + \ldots + h_1) = \varphi(h_m, \dots, h_1; x)$. Lemma 3 below shows that Γ is an *asymptotic pseudo-trajectory*, as defined by Benaim and Hirsch [**2**], that is,

$$\lim_{t\to\infty} |\Gamma(t+T) - \Phi(T, \Gamma(t))| = 0 \tag{9}$$

locally uniformly in $T > 0$. They prove in [**2**, Theorem 7.2 (i)] that the ω–limit set Λ of an asymptotic pseudo–trajectory with compact closure is *internally chain transitive*, that is, the restriction of Φ to Λ is a chain recurrent flow. (Λ is obviously nonempty, compact, connected and Φ-invariant.) Hence, Λ is contained in the chain recurrent set of the limiting ODE. For the convenience of the reader, we include an alternative proof below, see Lemma 4.

Recall that $\kappa(T) = K(e^{\gamma T} - 1)/\gamma$ and set, for $M = 1, 2, \cdots$, $h_{\max[1,M]} = \max\{h_1, \cdots, h_M\}$.

Lemma 3.

$$\left|\Phi\left(\sum_{k=1}^{M} h_k, x\right) - \varphi(h_M, \cdots, h_1; x)\right| \leqslant \kappa(T) h^p_{\max[1,M]} \qquad \text{for } x \in \mathbb{R}^n,\ 0 \leqslant \sum_{k=1}^{M} h_k \leqslant T.$$

PROOF:

$$
\begin{aligned}
&|\Phi(h_M,\Phi(h_{M-1},\cdots,\Phi(h_1,x)\cdots)) - \varphi(h_M,\varphi(h_{M-1},\cdots,\varphi(h_1,x)\cdots))| \\
&\leqslant \sum_{m=1}^{M}\left|\Phi\left(\sum_{i=m+1}^{M} h_i,\Phi(h_m,\varphi(h_{m-1},\cdots,h_1;x))\right) - \Phi\left(\sum_{i=m+1}^{M} h_i,\varphi(h_m,\varphi(h_{m-1},\cdots,h_1;x))\right)\right| \\
&\leqslant \sum_{m=1}^{M} |\Phi_x(h_M,\cdot)|\cdots|\Phi_x(h_{m+1},\cdot)|\cdot Kh_m^{p+1} \\
&\leqslant \sum_{m=1}^{M} (1+\gamma h_M)\cdots(1+\gamma h_{m+1})\cdot Kh_m h^p_{\max[1,M]} \\
&\leqslant \sum_{m=1}^{M} e^{\gamma(h_M+\cdots+h_{m+1})}\cdot Kh_m h^p_{\max[1,M]} = Kh^p_{\max[1,M]}e^{\gamma T}\sum_{m=1}^{M} e^{-\gamma(h_1+\cdots+h_m)}h_m.
\end{aligned}
$$

The remaining sum is the lower sum of an integral, as seen by transforming $s_k = \sum_{i=1}^{k} h_i$, and hence satisfies

$$= \sum_{m=1}^{M} e^{-\gamma s_m}(s_m - s_{m-1}) \leqslant \int_0^T e^{-\gamma s}ds = \frac{1-e^{-\gamma T}}{\gamma},$$

which completes the proof of Lemma 3. □

LEMMA 4. *The limit set of an asymptotic pseudotrajectory with compact closure is internally chain transitive for the original (limiting) flow.*

PROOF: Let p, q be two points in the limit set Λ of the asymptotic pseudotrajectory, and $\varepsilon, T > 0$ be given. We have to construct an (ε, T)–chain from p to q in Λ. Let $\delta < \varepsilon/4$ be such that $d(x,y) < \delta, y \in \Lambda, t \in [0,2T]$ implies $|\Phi(t,x) - \Phi(t,y)| < \varepsilon/2$.

The asymptotic pseudotrajectory (9) provides a (δ, T)–chain in $\mathbb{R}^n$, connecting p and q: $p = w_0, w_1, \ldots, w_N = z$, $t_i \in [T, 2T]$, such that

$$|w_{i+1} - \Phi(t_i, w_i)| \leqslant \delta.$$

We may furthermore assume that $d(w_i, \Lambda) < \delta$. Now pick any $q_i \in \Lambda$ with $d(w_i, q_i) < \delta$. Then

$$
\begin{aligned}
|q_{i+1} - \Phi(t_i,q_i)| &\leqslant |q_{i+1} - w_{i+1}| + |w_{i+1} - \Phi(t_i,w_i)| + |\Phi(t_i,w_i) - \Phi(t_i,q_i)| \\
&\leqslant \delta + \delta + \frac{\varepsilon}{2} < \varepsilon
\end{aligned}
$$

and (q_i) is the required (ε, T)–chain from p to q in Λ. □

THEOREM 4. *For each $x \in \mathbb{R}^n$, the ω–limit set*

$$\omega_{h_1,h_2,\dots}(x) = \bigcap_{N=1}^{\infty} \overline{\{\varphi(h_M,\cdots,h_1;x) : M \geqslant N\}}$$

is internally chain recurrent.

PROOF: Lemma 3, with $x = \Gamma(t)$, shows that Γ satisfies (9). Hence, by Lemma 4, $\omega(\Gamma)$ is internally chain recurrent. It is elementary to see that $\omega(\Gamma) = \omega_{h_1,h_2,\dots}(x)$. □

REMARKS. Observe, that the C^2–assumption on φ was not used in this section. The technique developed in section 2 gives easily, but only, that this ω–limit set is contained in $\mathcal{C}$.

Note, that our dissipativity assumption implies that $\omega_{h_1,h_2,\dots}(x)$ is compact but does not ensure that it is nonempty. However, there exists a positive constant $h(x)$ with the property that $\emptyset \neq \omega_{h_1,h_2,\dots}(x)$ whenever $h_{\max[1,\dots]} \leqslant h(x)$. Similarly, given a compact set $Q \subseteq \mathbb{R}^n$, there exists a positive constant $h(Q)$ such that $\emptyset \neq \omega_{h_1,h_2,\dots}(Q) = \bigcap_{N=1}^{\infty} \overline{\{\varphi(h_M,\cdots,h_1;Q) : M \geqslant N\}} \subset \mathcal{A}$ whenever $h_{\max[1,\dots]} \leqslant h(Q)$.

REFERENCES

[1] E. Akin, *The general topology of dynamical systems* (Amer. Math. Soc., Providence, R.I., 1993).

[2] M. Benaim and M.W. Hirsch, 'Asymptotic pseudotrajectories, chain recurrent flows and stochastic approximations', (preprint).

[3] W.J. Beyn, 'On invariant curves for one–step methods', *Numer. Math.* **51** (1987), 103–122.

[4] P. Brunovsky, J. Hofbauer and T. Nagylaki, 'Convergence of multilocus systems under weak epistasis or weak selection', (preprint).

[5] C. Conley, *Isolated invariant sets and the Morse index* (Amer. Math. Soc. CBMS 38, Providence, R.I., 1978).

[6] J.E. Franke and J.F. Selgrade, 'Hyperbolicity and chain recurrence', *J. Differential Equations* **26** (1977), 27–36.

[7] B.M. Garay, 'Discretization and normal hyperbolicity', *Z. Angew. Math. Mech.* **74** (1994), T 662–663.

[8] B. M. Garay, 'On C^j–closeness between the solution flow and its numerical approximation', *J. Difference Eq. Appl.* **2** (1996), 67–86.

[9] B.M. Garay, 'On structural stability of ordinary differential equations with respect to discretization methods', *Numer. Math.* **72** (1996), 449–479.

[10] M.C. Irwin, *Smooth dynamical systems* (Academic Press, New York, 1980).

[11] P. Kloeden and J. Lorenz, 'Stable attracting sets in dynamical systems and their one–step discretizations', *SIAM J. Numer. Anal.* **23** (1986), 986–995.

[12] C. Pugh, 'An improved closing lemma and a general density theorem', *Amer. J. Math.* **89** (1967), 1010–1021.

[13] A. Stuart, 'Numerical analysis of dynamical systems', *Acta Numerica* 4 (1994), 467–572.

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