Wintersemester 2000/2001
Analytic Number Theory I

Freitag 9:30 Uhr in 25.22-00.72

This lecture is an introduction to analytic number theory. It is accessible to anyone with knowledge of analysis, in particular complex function theory including complex integration and residue calculus.

The syllabus is as follows:

- **1 Arithmetical functions and Dirichlet Multiplication:**

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right), \quad \log n = \sum_{d|n} \Lambda(d)$$

- **2 Elementary Theorems on the Distribution of Prime Numbers**

$$\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$$

- **3 Dirichlet's Theorem on Primes in Arithmetic Progressions**

$$\sum_{p \le x,\, p \equiv h(k)} \frac{\log p}{p} = \frac{1}{\varphi(k)} \log x + O(1)$$

- **4 Dirichlet Series and Euler Products**

$$\zeta(s) = \prod_p \frac{1}{1 - \frac{1}{p^s}}$$

- **5 Zetafunctions and L-series**

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$\zeta(1-s) = 2(2\pi)^{-s} \Gamma(s) \cos\Big(\frac{\pi s}{2}\Big)\, \zeta(s)$$

- **6 Analytic Proof of the Prime Number Theorem**

$$\pi(x) \sim \frac{x}{\log x},$$

$$\pi(x) \sim \int_2^x \frac{dt}{\ln t},$$

$$\frac{\psi_1(x)}{x^2} = \frac{1}{2\pi i} \int_{c-\infty i}^{c+\infty i} \frac{x^{s-1}}{s(s+1)} \left(-\frac{\zeta'(s)}{\zeta(s)} \right) ds$$