Midterm exam

1. State and sketch the proof of the revenue equivalence theorem for the case of private values. Explain shortly all important assumptions.
2. Describe shortly how you would expect bidders to behave in an English auction for the case of private values. Let N be the number of bidders participating in the auction and suppose their values are independently drawn for a distribution $F(x)$ with density $f(x)$. Derive the expected payment of a bidder with value x in a symmetric equilibrium.
3. Consider an all-pay auction with $N = 2$ and values independently drawn from $F(x) = x^{1/2}$ over $x \in [0, 1]$.
   (a) Describe shortly the rules of the auction and use the revenue equivalence theorem to derive the symmetric equilibrium bidding strategy.
   (b) Compute directly the symmetric equilibrium bidding strategy for this case.
4. Consider now a first-price sealed-bid auction, again for the case $N = 2$ and $F(x) = x^{1/2}$ over $x \in [0, 1]$.
   (a) Derive the symmetric equilibrium bidding strategy assuming that bidders are risk neutral and compute expected revenues to the seller.
   (b) Suppose, alternatively, that bidders have utility $u(b, x) = \sqrt{x - b}$ if winning the object, where x is the bidder's value and b is the price paid if winning. Derive the symmetric equilibrium bidding strategy for this case and compute expected revenues to the seller. Compare and explain shortly to the result you obtained for risk-neutral bidders.