Midterm exam

1. Suppose you are participating in an auction with $N = 10$ bidders with i.i.d. private values exponentially distributed according to $F(x) = 1 - \exp(-0.02x)$ with $x \in [0, +\infty)$. Your value for the item is $x = 150$. The item goes to the highest bidder.

   (a) All bidders bid their true values. What is the probability that you win the auction if you submit bid $b = 150$?

   (b) All bidders bid half of their true values. What is the probability that you win the auction if you follow the same bidding strategy?

   (c) All bidders bid half of their true values. What is the probability that you win the auction if you submit bid $b = 100$?

2. Suppose you participate in an open English (ascending) auction with $N = 4$ bidders and i.i.d. private values uniformly distributed over $[0, 1]$. What is your expected payment in the auction if your value is $x = 0.2$?

3. Consider a symmetric first-price, sealed-bid auction with N bidders and i.i.d. uniformly distributed values over the interval $[0, 100]$.

   (a) Compute the equilibrium bidding strategy. What would be the equilibrium bid of a bidder with value $x = 25$ if $N = 5$?

   (b) Compute the expected revenue to the seller. What would be the expected revenue in an auction with $N = 5$ bidders?

   (c) Compute the distribution of revenues (price distribution of the auction). In an auction with $N = 5$ bidders, what is the probability that the seller receives revenue larger than 80?

4. State and explain the revenue equivalence theorem for the case of private values. What are the main assumptions? Derive the general expression of the expected revenue.