

Final exam

1. Consider an auction with two buyers and independent private values uniformly distributed in the interval $[0, 1]$.
 - (a) What is the expected revenue to the seller in a standard auction *without* reserve price where a buyer with value $x = 0$ has zero expected payment?
 - (b) Instead, what would be the expected revenue to the seller in a second-price, sealed-bid auction *with* reserve price $r = 1/2$?
 - (c) Consider now the following selling mechanism which is commonly used to sell a house when a renter occupant has the right to match the offer of a potential buyer. Buyer 1 can submit an offer $b \geq 1/2$, where $1/2$ is the seller's reserve price. After observing b , buyer 2 has the choice to match this offer and win the object at price b . If buyer 1 does not submit any offer, then buyer 2 can obtain the good at price $1/2$, if she wants. Buyer 1 gets the good at price equal to her offer b if she submits an admissible offer that is not matched by buyer 2.
 - i. If buyer 1 submits an admissible offer b , when should buyer 2 with value x_2 match this offer? What should buyer 2 do if buyer 1 does not submit any offer?
 - ii. What is the optimal behavior of buyer 1 if her value $x_1 < 1/2$? What is her optimal offer b^* if $x_1 \geq 1/2$, given the optimal reaction of buyer 2?
Hint: Note that $x - 2b < 0$ for $b > 1/2$ since $x \leq 1$.
 - iii. What will buyers do in the equilibrium you just obtained and what is the expected revenue to the seller? Compare to the revenues of a standard auction without reserve and the second-price auction with $r = 1/2$. Is this selling mechanism optimal? Is it efficient?
2. Advertising space is auctioned among two firms who are also competitors in the aftermarket. Winning the auction results in a profit increase of X_i to firm i coming from advertisement. If the competitor wins the ad, then firm i suffers a profit loss of $X_i/2$, due to the loss in market share. Assume $X_i \sim U[0, 1]$ and the value of profit increase is private information to each firm.
 - (a) If the ad is sold using a first-price auction, what are the equilibrium bidding strategies? What is the expected revenue to the seller of the ad?
 - (b) If the ad is allocated using the Vickrey, Clarke, Groves mechanism, what would be the associated payments? Would the seller get the same expected revenues as with the auction?
 - (c) Does an efficient, incentive compatible and individually rational mechanism exist in this case that also exactly balances the seller's budget? What mechanism would you suggest for this purpose?
3. Consider a common value auction with two bidders. Each bidder gets a signal on the value of the object. Signals are independently and uniformly distributed on the interval $[0, 1]$. The true value of the object, unknown to the bidders before the auction, is given by $V = 2x_1 - x_2$.
 - (a) Find the equilibrium bidding strategies for the first- and second-price auctions.
 - (b) Compute the associated equilibrium expected revenue for each of the two auction formats.